Package 'dst'

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Type Package

Title Using the Theory of Belief Functions

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Description Using the Theory of Belief Functions for evidence calculus. Basic probability assignments, or mass functions, can be defined on the subsets of a set of possible values and combined. A mass function can be extended to a larger frame. Marginalization, i.e. reduction to a smaller frame can also be done. These features can be combined to analyze small belief networks and take into account situations where information cannot be satisfactorily described by probability distributions.

License GPL $(>= 2)$

LazyData true

BugReports <https://github.com/RAPLER/dst-1/issues>

RoxygenNote 7.3.2

Depends R $(>= 3.5.0)$

Imports dplyr, ggplot2, tidyr, Matrix, methods, parallel, rlang, utils

Suggests igraph, knitr, rmarkdown, tidyverse, testthat

VignetteBuilder knitr

NeedsCompilation no

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addTobca *Add some elements of 0 mass to an existing basic chance assignment.*

Description

Given a previously defined basic chance assignment (bca), the user may want to add some elements of the set of possible values or some subsets, even if they have zero mass value. This feature is useful, for example, to examine the measure of plausibility of these elements or subsets of zero mass value.

Usage

addTobca(x, tt, f)

Arguments

Value

x The original basic chance assignment x augmented with the added subsets defined by tt.

Author(s)

Claude Boivin

Examples

```
y \le - bca(tt = matrix(c(1,0,0,1,1,1),nrow=2, byrow = TRUE),
m = c(0.6, 0.4), cnames = c("a", "b", "c"), idvar = 1)
addTobea(y, matrix(c(0,1,0,0,0,1, 0,1,1), nrow = 3, byrow = TRUE))x \le -\text{bca}(\text{tt} = \text{matrix}(c(0,1,1,1,1,0,1,1,1),\text{nrow=3},byrow = TRUE), m=c(0.2, 0.5, 0.3),
cnames = c("a", "b", "c"), idvar = 1)
xy <- dsrwon(x,y)
xy1 \leftarrow addTobca(nzdsr(xy), matrix(c(0,1,0,0,0,1), nrow = 2, byrow = TRUE))xy1
# add all singletons to a bca
addTobca(x, tt = diag(rep(1, ncol(x$tt) ) ) )
```
ads *The Captain's Problem.* ads*: Relation between variables Arrival (A), Departure delay (D) and Sailing delay (S)*

Description

This dataset is the tt matrix establishing the relation $A = D + S$, where $A = 0.6$, $D = 0.3$ and $S =$ 0:3. The subset made of all the triplets (a,d,s) of (A x D x S) where $a = d + s$ is true has a mass value of 1. To construct the tt matrix, we put the variables A, D, S side by side, as in a truth table representation. Each triplet of the subset is described by a row of the matrix as a vector of zeros and ones.

Usage

ads

Format

An integer matrix with 18 rows and 17 columns

 $[1, c(1,2)]$ value = 0, not used

[1, 3:17] Identification numbers of the three variables. Column 3 to 9: variable 1; column 10 to 13: variable 2; column 14 to 17: variable 3.

nospec identification number of the specification

m the value of the specification, a number between 0 and 1

- 6 1 if 6 is part of the specification, 0 otherwise
- 5 1 if 5 is part of the specification, 0 otherwise
- 4 1 if 4 is part of the specification, 0 otherwise
- 3 1 if 3 is part of the specification, 0 otherwise
- 2 1 if 2 is part of the specification, 0 otherwise
- 1 1 if 1 is part of the specification, 0 otherwise
- 0 1 if 0 is part of the specification, 0 otherwise

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Author(s)

Claude Boivin, Stat.ASSQ

Source

Almond, R.G. [1988] Fusion and Propagation in Graphical Belief Models. Computing Science and Statistics: Proceedings of the 20th Symposium on the Interface. Wegman, Edward J., Gantz, Donald T. and Miller, John J. (ed.). American Statistical Association, Alexandria, Virginia. pp 365–370.

bca *Basic chance assignment mass function*

Description

Function bca is used to define subsets of a finite set Θ of possible values and to assign their corresponding mass value.

The set Θ is called the frame of discernment. Each subset A of Theta with a positive mass value is called a focal element or a proposition. The associated mass value is a number of the $(0,1]$ interval, called "basic chance assignment" (the basic probability assignment of Shafer's book). All other subsets that have not received a positive mass value are assumed to have a mass value of zero.

Usage

```
bca(
  tt = NULL,m,
  qq = NULL,fzt = FALSE,include_all = FALSE,
  cnames = NULL,
  con = NULL,ssnames = NULL,
  idvar = NULL,
  infovar = NULL,
  varnames = NULL,
  valuenames = NULL,
  inforel = NULL
)
```
Arguments

tt Mandatory. A (0,1)-matrix or a boolean matrix. The number of columns must match the number of elements (values) of the frame of discernment Θ. Each row is a subset of $Θ$. The last row is the frame $Θ$, represented by a vector of 1's.

Details

There is two ways of defining the bca: a (0,1) matrix or a list of subsets labels.

Value

y An object of class bcaspec called a bca for "basic chance assignment":

- tt The table of focal elements. Rownames of the matrix of focal elements are generated from the column names of the elements of the frame. See [nameRows](#page-45-1) for details.
- qq Commonality functions from the frame of discernment to $[0, 1]$
- spec A two column matrix. First column contains numbers given to the subsets, 1 to nrow(tt). Second column contains the mass values of the subsets.
- con The measure of conflict.
- infovar The number of the variable and the size of the frame of discernment.
- varnames The name of the variable.
- valuenames A list of length 1 consisting of the name of the variable with the names of the elements of the frame of discernment (the column names of the tt matrix).
- ssnames A list of subsets names done from the column names of the tt matrix.
- inforel Set at 0. used in function [bcaRel](#page-9-1).

bcaNorm 7

Author(s)

Claude Boivin

References

- Shafer, G., (1976). A Mathematical Theory of Evidence. Princeton University Press, Princeton, New Jersey, p. 38: Basic probability assignment.
- Guan, J. W. and Bell, D. A., (1991). Evidence Theory and its Applications. Elsevier Science Publishing company inc., New York, N.Y., p. 29: Mass functions and belief functions

Examples

```
tt<- t(matrix(c(1,0,1,1),ncol = 2))
m < -c(.9,.1)cnames <- c("yes","no")
bca(tt, m)
bca(tt, m, cnames)
tt1<- t(matrix(c(1,0,1,1),ncol = 2))
colnames(tt1) \leq c("yes", "no")
m \leq c(.9, .1)bca(tt=tt1, m, idvar = 1)x \le -\text{bca}(\text{tt=matrix}(c(0,1,1,1,1,0,1,1,1),\text{nrow} = 3,byrow = TRUE), m = c(0.2, 0.5, 0.3), include_all = TRUE,
cnames = c("a", "b", "c"), idvar = 1)
y \le -\text{bca}(\text{tt=matrix}(c(1, 0, 0, 1, 1, 1), nrow = 2,byrow = TRUE), m = c(0.6, 0.4),
cnames = c("a", "b", "c"), varnames = "y", idvar = 1)
vacuous <- bca(matrix(c(1,1,1), nrow = 1), m = 1, cnames = c("a","b","c"), ssnames = c("a","b","c"))
```
bcaNorm *Computer norm between two basic chance assignment objects*

Description

Computer norm between two basic chance assignment objects

Usage

bcaNorm $(x, y, p = 1)$

Arguments

Value

a number of norm evaluation

Author(s)

Peiyuan Zhu

Examples

```
y1 \leq bca(tt = matrix(c(0,1,1,1,1,0,1,1,1),nrow = 3,byrow = TRUE), m = c(0.2, 0.5, 0.3),
cnames = c("a", "b", "c"),
varnames = "x", idvar = 1)
y2 \le - bca(tt = matrix(c(1,0,0,1,1,1),nrow = 2,
byrow = TRUE), m = c(0.6, 0.4),
cnames = c("a", "b", "c"),
varnames = "x", idvar = 1)
y1y2<-dsrwon(y1,y2)
bcaNorm(y1y2,y1)
```


Description

This utility function does a simple printing of a bca

Usage

bcaPrint(x)

Arguments

x A list of class bcaspec.

Value

A table of subsets with their associated mass. Subsets are identified by row names.

Author(s)

Claude Boivin

```
z \le bca(tt = matrix(c(0,1,1,1,1,0,1,1,1),nrow = 3,
byrow = TRUE), m = c(0.2, 0.5, 0.3),
cnames = c("a", "b", "c"), idvar = 1)
bcaPrint(z)
```


Print summary statistics of large mass functions

Usage

```
bcaPrintLarge(
  x,
  info_list = "all",
  num_top_mass = 10,
  cut_width_size = 10,
  cut\_width_m = 1e-05\mathcal{L}
```
Arguments

Value

- table of basic and more comprehensive statistics of subsets
- table of basic and more comprehensive statistics of masses
- table of basic and more comprehensive statistics of masses vs subsets

Author(s)

Peiyuan Zhu

Examples

```
if (requireNamespace("tidyverse", quietly = TRUE) ) {
 ## library(tidyverse)
x \le -\text{bca}(\text{tt} = \text{matrix}(c(1,1,0,1,1,1)), \text{ nrow} = 2, \text{ byrow} = \text{TRUE}), \text{ m} = c(0.8, 0.2), \text{ cnames} = c(1,2,3))bcaPrintLarge(x)
}
```
bcaRel *Representation of a mass function in a product space*

Description

This function is used to represent a relation between two or more variables in their product space P. The relation can be described by more than one subset of P. Each subset can also include more than one element. Complete disjunctive coding is used to represent one element in the input matrix of the function.

Usage

```
bcaRel(
  tt,
  spec,
  infovar,
  varnames,
  valuenames,
  relnb = NULL,
  infovarnames,
  infovaluenames
)
```
Arguments

bcaRel 11

Value

An object of class bcaspec called a bca for "basic chance assignment". This is a list containing the following components:

- con The measure of conflict.
- tt The resulting table of subsets. Rownames of the matrix of subsets are generated from the column names of the elements of the product frame. See [nameRows](#page-45-1) for details.
- spec The resulting two-column matrix of specification numbers with associated mass values.
- infovar The two-column matrix of variables number and size given in the input data.
- valuenames A list of the names of the variables with the name of the elements of their frame of discernment.
- inforel A two-column matrix containing the relation number and the depth (number of variables) of the relation.

Author(s)

Claude Boivin

```
# A logical implication rule
# A typical relation between two variables is the
# logical implication a -> b. Let us suppose
# that a stands for Rain: {yes, no} and b stands for
# Roadworks: {yes, no}. From experience,
# I am 75 % sure that there will be RoadWorks if there is no rain.
 # 1. The tt table of the logical implication
 ttrwf <- matrix(c(0,1,1,0,1,0,1,0,1,0,0,1,1,1,1,1),
 nrow = 4, byrow = TRUE,
 dimnames = list(NULL, c("rWdy", "rWdn", "Ry", "Rn")) )
 # 2. The mass distribution
 specrw <- matrix(c(1,1,1,2,0.75,0.75,0.75,0.25), ncol = 2,
 dimnames = list(NULL, c("specnb", "mass")))
 # 3. Variables numbers and sizes
 inforw \leq matrix(c(4,5,2,2), ncol = 2,
dimnames = list(NULL, c("varnb", "size")) )
bcaRel(t) = ttrwf, spec = specrw, infovar = inforw,
varnames = c("RdWorks", "Rain"), relnb = 6)
```


When working with large frames of discernment, the bca resulting of repeated application of Dempster's Rule of Combination can become big. One way to handle this situation could be to group subsets whose mass is less than a small treshold value. The function bcaTrunc serves this purpose to reduce a large bca to its main elements.

Usage

```
bcaTrunc(x, seuil, use_ssnames = FALSE)
```
Arguments

Value

tr_x The bca object truncated.

Author(s)

Claude Boivin

```
x \leftarrow bca(tt = matrix(c(0,1,0,0,0,0,1,1,
1,1,0,0,
1,0,1,0,
0,1,1,0,
1,1,1,1),ncol=4, byrow = TRUE), m = c(0.2, 0.5, 0.06, 0.04, 0.03, 0.17),
cnames = c("a", "b", "c", "d")bcaPrint(x)
tr_x < -bcaTrunc(x, seuil = 0.1)bcaPrint(tr_x)
```
belplau *Calculation of the degrees of Belief and Plausibility of a basic chance assignment (bca).*

Description

Degrees of Belief Bel and Plausibility Pl of the focal elements of a bca are computed. The ratio of the plausibility of a focal element against the plausibility of its contrary is also computed. Subsets with zero mass can be excluded from the calculations.

Usage

 $belplau(x, remove = FALSE, h = NULL, fzt = FALSE)$

Arguments

Details

The degree of belief Bel is defined by:

$$
bel(A) = Sum((m(B); B \subseteq A))
$$

for every subset B of A.

The degree of plausibility pl is defined by:

$$
pl(A) = Sum[(m(B); B \cap A \neq \emptyset]
$$

for every subset B of the frame of discernment. The plausibility ratio of a focal element A versus its contrary not A is defined by: $Pl(A)/(1 Bel(A)).$

Value

A matrix of M rows by 3 columns is returned, where M is the number of focal elements:

- Column 1: the degree of Belief bel;
- Column 2: the degree of Disbellief (belief in favor of the contrary hypothesis) disbel;
- Column 3: the degree of Epistemic uncertainty unc;
- Column 4: the degree of Plausibility plau;
- Column 5: the Plausibility ratio rplau.

Author(s)

Claude Boivin, Peiyuan Zhu

References

- Shafer, G., (1976). A Mathematical Theory of Evidence. Princeton University Press, Princeton, New Jersey, p. 39-43.
- Williams, P., (1990). An interpretation of Shenoy and Shafer's axioms for local computation. International Journal of Approximate Reasoning 4, pp. 225-232.

Examples

```
x \le -\text{bca}(\text{tt} = \text{matrix}(c(0,1,1,1,1,0,1,1,1),\text{nrow} = 3,byrow = TRUE), m = c(0.2, 0.5, 0.3),
cnames = c("a", "b", "c"), varnames = "x", idvar = 1)
belplau(x)
y \le - bca(tt = matrix(c(1,0,0,1,1,1),nrow = 2,
byrow = TRUE), m = c(0.6, 0.4),
cnames = c("a", "b", "c"), varnames = "y", idvar = 1)
xy <- nzdsr(dsrwon(x,y))
belplau(xy)
print("compare all elementary events")
xy1 \leq - \text{addTobca}(x = xy, tt = matrix(c(0,1,0,0,0,1), nrow = 2, byrow = TRUE))belplau(xy1)
belplau(xy1, remove = TRUE)belplau(xy1, h = matrix(c(1, 0, 0, 0, 1, 1), nrow = 2, byrow = TRUE))
```
belplauEval *Evaluate A, B errors*

Description

Calculate error A, B, and total error A+B by comparing two vectors as defined below. One vector represents the truth and the other represents a numerical quantity of importance.

- Error A: out of all the comparisons between two elements, what proportion of errors are due to indicating an irrelevant element as more important than a relevant element
- Error B: out of all the comparisons between two elements, what proportion of errors are due to indicating a relevant element as less important than an irrelevant element
- Total error A+B: the sum of quantity A and quantity B

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Usage

```
belplauEval(
  belplau_mat,
  true_order,
  var = "rplau",err = "A",is_belplau = TRUE
\mathcal{L}
```
Arguments

Value

A number in $[0, 1]$ of error A, B, or total error A+B.

Author(s)

Peiyuan Zhu

```
x \le -\text{bca}(\text{tt} = \text{matrix}(c(0,1,1,1,1,0,1,1,1),\text{nrow} = 3,byrow = TRUE), m = c(0.2, 0.5, 0.3),
cnames = c("a", "b", "c"), varnames = "x", idvar = 1)
belplau(x)
y \le - bca(tt = matrix(c(1,0,0,1,1,1),nrow = 2,
byrow = TRUE), m = c(0.6, 0.4),
cnames = c("a", "b", "c"), varnames = "y", idvar = 1)
xy <- nzdsr(dsrwon(x,y))
z<-belplau(xy,h=ttmatrixPartition(xy$infovar[2],xy$infovar[2]))
belplauEval(z,c(0,1,0))
```


Calculate belief, disbelief, unknown, plausibility, plausibility ratio

Usage

belplauH(MACC, W2, h)

Arguments

Value

A matrix of M rows by 5 columns is returned, where M is the number of hypothesis tested:

- Column 1: the degree of Belief bel;
- Column 2: the degree of Disbellief (belief in favor of the contrary hypothesis) disbel;
- Column 3: the degree of Epistemic uncertainty unc;
- Column 4: the degree of Plausibility plau;
- Column 5: the Plausibility ratio rplau.

Author(s)

Peiyuan Zhu

```
x \le -\text{bca}(\text{tt} = \text{matrix}(c(1,1,0,1,1,1)), \text{ nrow} = 2, \text{ byrow} = \text{TRUE}), m = c(0.8, 0.2), \text{ cameras} = c(1,2,3))belplauH(MACC = x$spec[, 2], W2 = x$tt, h = x$tt)hyp <- matrix(c(0,1,0, 0,1,1), nrow = 2, byrow = TRUE)
rownames(hyp) <- nameRows(hyp)
belplauH(MACC = x$spec[, 2], W2 = x$tt, h = hyp)
```
belplauHLogsumexp *Calculate belief, disbelief, unknown, plausibility, plausibility ratio with logsumexp*

Description

Calculate belief, disbelief, unknown, plausibility, plausibility ratio with logsumexp

Usage

belplauHLogsumexp(MACC, W2, h)

Arguments

Value

A matrix of M rows by 5 columns is returned, where M is the number of hypothesis tested:

- Column 1: the degree of Belief bel;
- Column 2: the degree of Disbellief (belief in favor of the contrary hypothesis) disbel;
- Column 3: the degree of Epistemic uncertainty unc;
- Column 4: the degree of Plausibility plau;
- Column 5: the Plausibility ratio rplau.

Author(s)

Peiyuan Zhu

```
x \le -\text{bca}(\text{tt} = \text{matrix}(c(1,1,0,1,1,1), \text{ nrow} = 2, \text{ byrow} = \text{TRUE}), m = c(0.8, 0.2), \text{ cnames} = c(1,2,3))belplauH(MACC = x$spec[, 2], W2 = x$tt, h = x$tt)hyp <- matrix(c(0, 1, 0, 0, 1, 1)), nrow = 2, byrow = TRUE)
rownames(hyp) <- nameRows(hyp)
belplauH(MACC = x$spec[, 2], W2 = x$tt, h = hyp)
```


Compute belief, disbelief, unknown, plausibility, plausibility ratio based on commonality function

Usage

belplauHQQ(qq, h = NULL)

Arguments

Value

z A matrix of M rows by 5 columns is returned, where M is the number of hypothesis tested:

- Column 1: the degree of Belief bel;
- Column 2: the degree of Disbellief (belief in favor of the contrary hypothesis) disbel;
- Column 3: the degree of Epistemic uncertainty unc;
- Column 4: the degree of Plausibility plau;
- Column 5: the Plausibility ratio rplau.

Author(s)

Peiyuan Zhu

```
x \le -\text{bca}(\text{tt} = \text{matrix}(c(0,1,1,1,1,0,1,1,1),\text{nrow} = 3, \text{byrow} = \text{TRUE}),m = c(0.2, 0.5, 0.3), cnames = c("a", "b", "c"), varnames = "x", idvar = 1)
qq <- commonality(x$tt,x$spec[,2])
belplauHQQ(qq,h=matrix(c(0,1,0), nrow=1, byrow=TRUE))
```
belplauLogsumexp *Calculation of the degrees of Belief and Plausibility of a basic chance assignment (bca) with logsumexp.*

Description

Degrees of Belief Bel and Plausibility Pl of the focal elements of a bca are computed. The ratio of the plausibility of a focal element against the plausibility of its contrary is also computed. Subsets with zero mass can be excluded from the calculations.

Usage

belplauLogsumexp(x, remove = FALSE, $h = NULL$, $fzt = FALSE$)

Arguments

Details

The degree of belief Bel is defined by:

$$
bel(A) = Sum((m(B); B \subseteq A))
$$

for every subset B of A.

The degree of plausibility pl is defined by:

$$
pl(A) = Sum[(m(B); B \cap A \neq \emptyset]
$$

for every subset B of the frame of discernment. The plausibility ratio of a focal element A versus its contrary not A is defined by: $Pl(A)/(1 Bel(A)).$

Value

A matrix of M rows by 3 columns is returned, where M is the number of focal elements:

- Column 1: the degree of Belief bel;
- Column 2: the degree of Disbellief (belief in favor of the contrary hypothesis) disbel;
- Column 3: the degree of Epistemic uncertainty unc;
- Column 4: the degree of Plausibility plau;
- Column 5: the Plausibility ratio rplau.

Author(s)

Claude Boivin, Peiyuan Zhu

References

- Shafer, G., (1976). A Mathematical Theory of Evidence. Princeton University Press, Princeton, New Jersey, p. 39-43.
- Williams, P., (1990). An interpretation of Shenoy and Shafer's axioms for local computation. International Journal of Approximate Reasoning 4, pp. 225-232.

Examples

```
x \le -\text{bca}(\text{tt} = \text{matrix}(c(0,1,1,1,1,0,1,1,1),\text{nrow} = 3,byrow = TRUE), m = c(0.2, 0.5, 0.3),
cnames = c("a", "b", "c"), varnames = "x", idvar = 1)
belplau(x)
y \le - bca(tt = matrix(c(1,0,0,1,1,1),nrow = 2,
byrow = TRUE), m = c(0.6, 0.4),
cnames = c("a", "b", "c"), varnames = "y", idvar = 1)
xy <- nzdsr(dsrwon(x,y))
belplau(xy)
print("compare all elementary events")
xy1 \leq - addTobca(x = xy, tt = matrix(c(0,1,0,0,0,1), nrow = 2, byrow = TRUE))
belplau(xy1)
belplau(xy1, remove = TRUE)
belplau(xy1, h = matrix(c(1, 0, 0, 0, 1, 1), nrow = 2, byrow = TRUE))
```
belplauPlot *Plot belplau matrix*

Description

Plot belplau matrix

Usage

```
belplauPlot(
  belplau_mat,
  xlab,
  color,
 y = "rplau",x = "index",levels = NULL,
  legend\_title = "".main\_title = "",is_log_scale = TRUE,
  is_negative = FALSE,
```

```
is_factor = FALSE
\mathcal{L}
```
Arguments

Value

a plot of a column of the belplau matrix or a numerical vector quantifying order of importance of the elements of the frame

Author(s)

Peiyuan Zhu

Examples

```
bpa <- bca(tt = matrix(c(0,1,1,1,1,0,1,1,1),nrow = 3,
byrow = TRUE), m = c(0.2, 0.5, 0.3),
cnames = c("a", "b", "c"), varnames = "x", idvar = 1)
bel_plau <- belplau(bpa)
belplauPlot(bel_plau, c("a","b","c"), c(1,3,2))
```
captain_result *The Captain's Problem.* swr*: Result of the evaluation of the Hypergraph at node Arrival (A)*

Description

This dataset is the tt bca resulting from the combination of the relations of the hypergraph and marginalization at node Arrival (A).

Usage

captain_result

Format

A list of 8 elements, of class bcaspec.

Author(s)

Claude Boivin, Stat.ASSQ

Source

Almond, R.G. [1988] Fusion and Propagation in Graphical Belief Models. Computing Science and Statistics: Proceedings of the 20th Symposium on the Interface. Wegman, Edward J., Gantz, Donald T. and Miller, John J. (ed.). American Statistical Association, Alexandria, Virginia. pp 365–370.

commonality *Compute qq from tt*

Description

qq is the commonality function as a set function from the subsets of the frame to $[0, 1]$. To evaluate it, input a set encoded in binary vector, so the commonality number at that set can be returned.

Usage

commonality(tt, m, fzt = FALSE)

Arguments

Value

f Commonality function

Author(s)

Peiyuan Zhu

```
x <- bca(tt = matrix(c(0,1,1,1,1,0,1,1,1),nrow = 3, byrow = TRUE),
m = c(0.2, 0.5, 0.3), cnames = c("a", "b", "c"), varnames = "x", idvar = 1)
qq <- commonality(x$tt,x$spec[,2])
qq(c(1,0,0))
```


The aplDecode function of the project APL in R (<https://rpubs.com/deleeuw/158476>) has been adapted to follow the standard implementation of the APL decode function.

Usage

decode(base, ind)

Arguments

Details

If the base value is a number system, e.g. base 2, we need only to enter it as a scalar, which is then processed to match the length of the expression to decode. If length(ind) is less than length(base), zeroes are added to the left of the vector ind to match the length of the two vectors. And vice-versa.

Value

A scalar representing the conversion of the coded number ind to its decimal representation.

Author(s)

Claude Boivin

References

- Jan de Leeuw and Masanao Yajima (March 07, 2016) *APL in R (Version 009)*, Source code. <https://rpubs.com/deleeuw/158476>
- L. Gilman and A. J. Rose.(1974): *APL an Interactive Approach*, Second Edition, John Wiley, New York.
- APL 68000 Level II language manual. MicroAPL Ltd. 1990.

```
decode(c(2,2,2,2), c(1,0,1,1)) # Find the base 10 value of the base 2 number 1011.
decode(2, c(1,0,1,1)) # left argument is extended to vector c(2,2,2,2)decode(c(365,24,60), c(2,1,57)) # transform 2 days 1 h 57 min in minutes
decode(c(365,24,60), c(1,57)) # right vector extended
decode(c(24, 60), c(2, 1, 57)) # left vector extended
decode(1.5, c(1,2,3)) # polynomial 1*x^2 +2*x +3 evaluated at x=1.5
```


This dataset is the tt matrix establishing the relation between the four variables. Each event (loading $=$ true, forecast $=$ foul, Maintenance $=$ true) adds one day of Departure Delay. The elements (d,l, f, m) of $(D \times L \times F \times M)$ satisfying the relation form a subset with a mass value of 1. To construct the tt matrix, we put the variables D,L,F,M side by side, as in a truth table representation. Each 4-tuple of the subset is described by a row of the matrix as a vector of zeros and ones.

Usage

dlfm

Format

An integer matrix with 10 rows and 12 columns.

 $[1,\mathbf{c}(1,2)]$ value = 0, not used

[1,3:12] Identification numbers of the four variables. Column 3 to 6: variable 2; columns 7,8: variable 4; columns 9, 10: variable 5: columns 11,12: variable 6.

nospec identification number of the specification

m the value of the specification, a number between 0 and 1

3 1 if d3 is part of the specification, 0 otherwise

2 1 if d2 is part of the specification, 0 otherwise

1 1 if d1 is part of the specification, 0 otherwise

0 1 if d0 is part of the specification, 0 otherwise

true 1 if true is part of the specification, 0 otherwise

false 1 if false is part of the specification, 0 otherwise

foul 1 if foul is part of the specification, 0 otherwise

fair 1 if fair is part of the specification, 0 otherwise

DoSSnames 25

Author(s)

Claude Boivin, Stat.ASSQ

Source

Almond, R.G. [1988] Fusion and Propagation in Graphical Belief Models. Computing Science and Statistics: Proceedings of the 20th Symposium on the Interface. Wegman, Edward J., Gantz, Donald T. and Miller, John J. (ed.). American Statistical Association, Alexandria, Virginia. pp 365–370.

DoSSnames *Construct subsets names from column names of a tt matrix*

Description

Construct subsets names from column names of a tt matrix

Usage

DoSSnames(tt)

Arguments

tt A description matrix with column names

Value

subsets_names A list of names.

Author(s)

Claude Boivin

```
y1 \leftarrow bca(tt = matrix(c(0,1,1,1,1,0,1,1,1), nrow = 3,byrow = TRUE), m = c(0.2, 0.5, 0.3),
cnames = c("a", "b", "c"),
varnames = "x", idvar = 1)
DoSSnames(y1$tt)
```
The generalized inner product of two matrices combines two operators in the same manner as the classical inner product defined for the multiplication of two matrices. The number of rows of the second matrix must be equal the number of columns of the first matrix.

Usage

dotprod(x, y, g, f)

Arguments

Value

The result of the generalized inner product is returned.

Author(s)

Claude Boivin

```
print("Standard matrix product")
x \le -y \le - matrix(c(1:6), nrow = 2, byrow = TRUE)
dotprod(x, t(y), g = " +", f = " *") ## same as x %*% t(y)
print("Find some data x2 in the rows of a larger matrix y2")
x2 \le matrix(c(1,0,0,1,1,1), nrow = 2, byrow = TRUE)
y2 <- matrix(c(1,0,0,0,1,0,1,1,0,0,1,1,1,1,1),
nrow = 5, byrow = TRUE)
(1: nrow(y2)) * dotprod(x2, t(y2), g = "&", f = "==")print("Find some names in a long list")
team_names <- matrix(c("Patrick", "Dole", "Amanda",
 "Dole", "Robert", "Calvin", "Alvina", "Klein",
  "Robert", "Gariepy", "Nellie", "Arcand"),
   ncol = 2, byrow = TRUE)
colnames(team_names) <- c("First_name", "Last_name")
print("Where in the list are the person with first name Robert and where are the Doles?")
BobandDoles <- matrix(c("Robert", "", "", "Dole"),
 ncol = 2, byrow = TRUE)
dotprod(team_names, t(BobandDoles),g="|",f="==") * (1:nrow(team_names))
```


Recursive function.

Usage

doubles(x)

Arguments

x A matrix of numeric, character or logical type.

Value

The submitted matrix with duplicated rows removed from.

Author(s)

Claude Boivin

Examples

```
td0 \leq -\text{matrix}(c(rep(c(1,0,1),times=3),0,0,1,1,1,1, 1,1,1),ncol = 3,byrow = TRUE)(doubles(td0))
td1 <- matrix(c(rep(c(1,0,1),times=3),0,0,1,1,1,1),ncol = 3,byrow = TRUE)
(doubles(td1))
td2 \leq \text{matrix}(c(1:3, 1:3, 4:6, 1:3), nrow = 4, byrow = TRUE)(doubles(td2))
td3 <- matrix(c("d","e","f", rep(c("a","b","cc"),times = 3),"g","h","i"),nrow = 5,byrow = TRUE)
(doubles(td3))
td4 <- matrix(as.logical(td1),nrow = 5,byrow = TRUE)
(doubles(td4))
```
dsrwon *Combination of two mass functions*

Description

The unnormalized Dempster's rule is used to combine two mass functions mx and my defined on the same frame of discernment and described by their respective basic chance assignments x and y. Dempster's rule of combination is applied. The normalization is not done, leaving the choice to the user to normalize the results or not (for the normalization operation, see function [nzdsr](#page-46-1)).

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Usage

```
dsrwon(
  x,
 y,
 mcores = "no",
 use_ssnames = FALSE,
 use_qq = FALSE,varnames = NULL,
  relnb = NULL,skpt_tt = FALSE,
  infovarnames
)
```
Arguments

Details

The calculations make use of multiple cores available.

The two bca's x and y must be defined on the same frame of discernment for the combination to take place. The relation number of the x input is given to the output result.

Value

A basic chance assignment with these two components added:

- I12 Intersection table of subsets.
- Sort_order Sort order of subsets.

Author(s)

Claude Boivin, Peiyuan Zhu

References

Shafer, G., (1976). A Mathematical Theory of Evidence. Princeton University Press, Princeton, New Jersey, pp. 57-61: Dempster's rule of combination.

dsrwonLogsumexp 29

Examples

```
y1 \le -\text{bca}(\text{tt} = \text{matrix}(c(0,1,1,1,1,0,1,1,1),\text{nrow} = 3,byrow = TRUE), m = c(0.2, 0.5, 0.3),
cnames = c("a", "b", "c"),varnames = "x", idvar = 1)
y2 \le - bca(tt = matrix(c(1,0,0,1,1,1),nrow = 2,
byrow = TRUE), m = c(0.6, 0.4),
cnames = c("a", "b", "c"),
varnames = "x", idvar = 1)
dsrwon(y1,y2)
# Sparse matrices
y1s \leftarrow y1y2s <- y2
y1s$tt <- methods::as(y1$tt, "RsparseMatrix")
y2s$tt <- methods::as(y2$tt, "RsparseMatrix")
y1y2s <- dsrwon(y1s, y2s, use_ssnames = TRUE)
# using commonalities
bma \leq bca(tt=matrix(c(1,1,0,1,rep(1,4)), ncol = 4, byrow = TRUE),
m = c(0.1, 0.9), cnames = c("a", "b", "c", "d")bma2 <- dsrwon(bma, bma, use_qq = TRUE)
vacuous \leq bca(matrix(c(1,1,1), nrow = 1), m = 1, cnames = c("a","b","c"))
dsrwon(vacuous, vacuous)
```
dsrwonLogsumexp *Combination of two mass functions with logsumexp*

Description

The unnormalized Dempster's rule is used to combine two mass functions mx and my defined on the same frame of discernment and described by their respective basic chance assignments x and y. Dempster's rule of combination is applied. The normalization is not done, leaving the choice to the user to normalize the results or not (for the normalization operation, see function [nzdsr](#page-46-1)).

Usage

```
dsrwonLogsumexp(
  x,
 y,
  mcores = "no",
 use_ssnames = FALSE,
 use_{qq} = FALSE,
  varnames = NULL,
  relnb = NULL,skpt_t = FALSE,
  infovarnames
)
```
Arguments

Details

The calculations make use of multiple cores available.

The two bca's x and y must be defined on the same frame of discernment for the combination to take place. The relation number of the x input is given to the output result.

Value

A basic chance assignment with these two components added:

- I12 Intersection table of subsets.
- Sort order Sort order of subsets.

Author(s)

Claude Boivin, Peiyuan Zhu

References

Shafer, G., (1976). A Mathematical Theory of Evidence. Princeton University Press, Princeton, New Jersey, pp. 57-61: Dempster's rule of combination.

```
y1 \leftarrow bca(tt = matrix(c(0,1,1,1,1,0,1,1,1), nrow = 3,byrow = TRUE), m = c(0.2, 0.5, 0.3),
cnames = c("a", "b", "c"),varnames = "x", idvar = 1)
y2 \le - bca(tt = matrix(c(1,0,0,1,1,1),nrow = 2,
byrow = TRUE), m = c(0.6, 0.4),
{\text{cnames}} = c("a", "b", "c"),varnames = "x", idvar = 1)
dsrwonLogsumexp(y1,y2)
# Sparse matrices
y1s \leq -y1
```


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```
y2s \leftarrow y2y1s$tt <- methods::as(y1$tt, "RsparseMatrix")
y2s$tt <- methods::as(y2$tt, "RsparseMatrix")
y1y2s <- dsrwonLogsumexp(y1s, y2s, use_ssnames = TRUE)
vacuous \leq bca(matrix(c(1,1,1), nrow = 1), m = 1, cnames = c("a","b","c"))
dsrwonLogsumexp(vacuous, vacuous)
```
elim *Reduction of a relation*

Description

This function works on a relation defined on a product of two variables or more. Having fixed a variable to eliminate from the relation, the reduced product space is determined and the corresponding reduced bca is computed.This operation is also called "marginalization".

Usage

elim(rel, xnb)

Arguments

Value

r The reduced relation

Author(s)

Claude Boivin

```
# We construct a relation between two variables to show marginalization.
wr_t t \leq \text{matrix}(c(1, rep(0,3), rep(c(1,0),3), 0, 1, 1, 1, 0, 0,1,0, rep(1,5),0,1,1,0, rep(1,5), ncol = 4, byrow = TRUE)
colnames(wr_tt) <- c("Wy Ry", "Wy Rn", "Wn Ry", "Wn Rn")
rownames(wr_tt) <- nameRows(wr_tt)
wr_spec = matrix(c(1:8, 0.017344, 0.046656,
0.004336, 0.199456,0.011664,0.536544,0.049864, 0.134136),
ncol = 2, dimnames = list(NULL, c("specnb", "mass")))
wr_infovar = matrix(c(4,5,2,2), ncol = 2,dimnames = list(NULL, c("varnb", "size")) )
wr_{rel} \leftarrow list(tt = wr_tt, con = 0.16, spec=wr_spec,
 infovar = wr_infovar, varnames = c("Roadworks","Rain"),
 valuenames = list( RdWorks = c("Wy", "Wn"), Rain=c("Ry", "Rn") ))
class(wr_rel) <- "bcaspec"
```

```
bcaPrint(elim(wr_rel, xnb = 5))
bcaPrint(elim(wr_rel, xnb = 4))
```
encode *Convert a value to its representation in another chosen base*

Description

The aplEncode function of the project APL in R (<https://rpubs.com/deleeuw/158476>) has been adapted to follow the standard implementation of the APL encode function.

Usage

encode(base, ind)

Arguments

Value

A vector or a matrix of the data converted.

Author(s)

Claude Boivin

References

- Jan de Leeuw and Masanao Yajima (March 07, 2016) *APL in R (Version 009)*, Source code. <https://rpubs.com/deleeuw/158476>
- L. Gilman and A. J. Rose.(1974): *APL an Interactive Approach*, Second Edition, John Wiley, New York.
- APL 68000 Level II language manual. MicroAPL Ltd. 1990.

```
encode(c(2, 2, 2, 2), 11) # find the base 2 representation of number 11
encode(c(365, 24, 60), 2997) # convert 2997 minutes to days-hrs-min.
```


This function works on a basic chance assignment (bca) x defined on a single variable. Iy= t Allows the addition of new values to the frame of discernment.

Usage

extFrame(x, use_ssnames = FALSE, lab = NULL)

Arguments

Value

zxtnd The bca with its frame extended

Author(s)

Claude Boivin

Examples

```
s1_e1 \leftarrow bca(tt = matrix(c(1, 0, 1, 1), nrow = 2, byrow = TRUE),m = c(0.6, 0.4), cnames = c("S1","S2"), varnames = "v1", idvar = 1)
s13_names <- extFrame(s1_e1, lab = "S3", use_ssnames =TRUE)
s13 \leftarrow extFrame(s1_e1, lab = "S3")
```
extmin *Extension of a relation*

Description

This function works on a basic chance assignment (bca) x defined on a single variable or more. A relation of reference is given, and an extension of the space of x is made to the larger product space of the relation of reference. The basic chance assignment to extend and the relation of reference must have at least one common variable for the extension to occur.

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Usage

extmin(rel1, relRef)

Arguments

Details

The relRef parameter is used to extract all the information on the variables, namely their identification numbers and the number of elements of each variable, variables names and columns names of the tt matrix. The relation of reference relRef may be a relation already existing or simply the the vacuous relation defined on the product set of variables of interest.

Value

the resulting extended bca.

Author(s)

Claude Boivin

References

G. Shafer and P. P. Shenoy. Local Computations in Hypertrees. School of Business, University of Kansas, Lawrence, KS, 1991. See p. 78, vacuous extension of a belief function.

```
# Making a vacuous reference relation and extending a bca to its space.
init\_tt = matrix(rep(1,10)), nrow = 1,
dimnames = list(NULL, c("3", "2", "1", "0",
"true", "false", "foul", "fair", "true", "false")) )
init_spec <- matrix(c(1,1), c_0) = 2,
dimnames = list(NULL, c("specnb", "mass")))
 init_info \leftarrow matrix(c(3, 4, 7, 8, 4, 2, 2, 2), ncol = 2,dimnames = list(NULL, c("varnb", "size")) )
relRef <- bcaRel(tt = init_tt, spec = init_spec,
 infovar = init_info,
 varnames = c("Sail", "Loading", "Weather", "Repairs"),
 relnb = 0)# a bcaspec defined on one variable
 l_{rel} \leftarrow bca(t) = matrix(c(1, 0, 1, 0, 1, 1), ncol = 2),m = c(0.3, 0.5, 0.2), cnames = c("true", "false"),
 infovar = matrix(c(4,2), ncol = 2,dimnames = list(NULL, c("varnb", "size"))),
 varnames = c("Loading"),
```

```
inforel = matrix(c(7,1), ncol = 2,dimnames = list(NULL, c("relnb", "depth"))))
z <- extmin(l_rel, relRef)
prmatrix(t(z$tt), collab = rep("", nrow(z$tt)))
```
fw *The Captain's Problem.* fw*: Relation between variables Forecast of the weather (F) and Weather at sea (W)*

Description

This dataset is the tt matrix establishing the relation between the two variables. An accurate forecast is described by this subset of two event: (Forecast = foul, Weather = foul) and (Forecast = fair, Weather = fair). We assign a mass value of 0.8 to this subset. The remaining mass of 0.2 is allotted to the frame. To construct the tt matrix, we put the variables F and W side by side, as in a truth table representation. Each pair of the subset is described by a row of the matrix as a vector of zeros and ones.

Usage

fw

Format

An integer matrix with 4 rows and 6 columns.

 $[1,\text{c}(1,2)]$ value = 0, not used

[1,3:6] Identification numbers of the two variables. Column 3,6: variable 5; columns 5,6: variable 7.

nospec identification number of the specification

m the value of the specification, a number between 0 and 1

foul 1 if foul is part of the specification, 0 otherwise

fair 1 if fair is part of the specification, 0 otherwise

Author(s)

Claude Boivin

Source

Almond, R.G. [1988] Fusion and Propagation in Graphical Belief Models. Computing Science and Statistics: Proceedings of the 20th Symposium on the Interface. Wegman, Edward J., Gantz, Donald T. and Miller, John J. (ed.). American Statistical Association, Alexandria, Virginia. pp 365–370.

Function inters returns a table of the intersection between two (0,1) or boolean matrices or two vectors. The two matrices must have the same number of columns. The two vectors must be of the same length. This function generalizes the intersection of two subsets represented by boolean vectors to the intersection of two matrices of subsets.

Usage

inters(x, y)

Arguments

Value

The result is a $(0,1)$ -table of dimensions $(M \times K) \times N$). In the case of vectors, the result is a $(0,1)$ table of dimensions $(1 \times K) \times 1$

Author(s)

Claude Boivin

```
mx \le - matrix(c(0,1,0,0,1,1,1,1,1,1),nrow = 3, byrow = TRUE, dimnames = list(NULL, c("a", "b", "c")))
rownames(mx) <- nameRows(mx)
my \le -matrix(c(\emptyset, \emptyset, 1, 1, 1, 1)), nrow = 2, byrow = TRUE, dimnames = list(NULL, c("a", "b", "c")))
rownames(my) <- nameRows(my)
inters(mx,my)
b1 <- matrix(c(FALSE, TRUE, TRUE), nrow=1)
b2 <- matrix(c(TRUE, TRUE, FALSE), nrow=1)
colnames(b1) <- colnames(b2) <- c("c1","c2","c3")
inters(b1,b2)
x3 <-matrix(c(1,1,0,1), ncol = 2, dimnames = list(NULL, c("a","b")))
y3 <-matrix(c(0,1,1,1), ncol = 2, dimnames = list(NULL, c("a","b")))
inters(x3,y3)
x4 \le -\text{matrix}(c(1,0,1,1,1,1,1,1),\text{now} = 2, \text{ byrow} = \text{TRUE}, \text{ dimnames} = \text{list(NULL}, c("a", "b", "c", "d"))y4 \le -\text{matrix}(c(1,0,0,1,1,1,1,1),\text{now} = 2, \text{byrow} = \text{TRUE}, \text{dimnames} = \text{list(NULL}, c("a", "b", "c", "d"))inters(x4,y4)
# Sparse matrices
stt1 <- Matrix::sparseMatrix(i= c(1,1,2,2,3,3,3), j= c(2,3,1,2,1,2,3), x = 1, dims = c(3,3))
```
intersBySSName 37

```
y1 \le - bca(tt = stt1, m = c(0.2,0.5, 0.3),
         cnames = c("a", "b", "c"),varnames = "x", idvar = 1)
stt2 <- Matrix::sparseMatrix(i= c(1,2,2,2), j= c(1,1,2,3), x = 1, dims = c(2,3))
y2 \le - bca(tt = stt2, m = c(0.6, 0.4),
         cnames = c("a", "b", "c"),varnames = "x", idvar = 1)
 sr <-inters(y1$tt, y2$tt)
 sr
 class(sr)
```
intersBySSName *Intersect two vectors of ssnames*

Description

Intersect two vectors of ssnames

Usage

intersBySSName(zx, yz)

Arguments

Value

ssnames in the intersection of the two bcas

Author(s)

Peiyuan Zhu

```
y1 \le -\text{bca}(\text{tt} = \text{matrix}(c(0,1,1,1,1,0,1,1,1),\text{nrow} = 3,byrow = TRUE), m = c(0.2, 0.5, 0.3),
cnames = c("a", "b", "c"),
varnames = "x", idvar = 1)
y2 \le - bca(tt = matrix(c(1,0,0,1,1,1),nrow = 2,
byrow = TRUE), m = c(0.6, 0.4),
cnames = c("a", "b", "c"),varnames = "x", idvar = 1)
intersBySSName(y1$ssnames[[1]], y2$ssnames[[2]])
```


Adding small probabilities

Usage

logsum(l1, l2)

Arguments

Value

sum of probabilities $exp(11)+exp(12)$

Author(s)

Peiyuan Zhu

Examples

sum of two 1e-5 exp(logsum(log(1e-5),log(1e-5)))

marrayToMatrix *Transformation of an array data to its matrix representation*

Description

The array representation or product space representation is converted to the matrix representation of the corresponding relation.

Usage

marrayToMatrix(mtt)

Arguments

mtt The matrix tt of the relation in array format

matrixToMarray 39

Value

The matrix representation of the data.

Author(s)

Claude Boivin

Examples

```
mtt <- array(c(1,0,0,0,1,1,0,0,1,0,0,1,1,0,1,0,1,1,0,1,1,1,1,0,1,0,1,1,1,1,1,1), c(2,2,8),
dimnames = list( RdWorks=c("Wy", "Wn") , Rain = c("Ry", "Rn"), ev=1:8) )
print(z <- marrayToMatrix(mtt))
```
matrixToMarray *Transformation of the tt matrix of a relation*

Description

The matrix representation of a relation is converted to the array representation or product space representation.

Usage

matrixToMarray(tt, valuenames)

Arguments

Value

mtt The array (product space) representation of the tt matrix.

Author(s)

Claude Boivin

Examples

```
# Define wr_tt, a matrix describing the relation between two variables
wr_t t \leq \text{matrix}(c(1, rep(0,3), rep(c(1,0),3),0,1,1,1,0,0,1,0,rep(1,5),0,1,1,0,rep(1,5)), ncol=4, byrow = TRUE)
colnames(wr_tt) <- c("Wy Ry", "Wy Rn", "Wn Ry", "Wn Rn")
rownames(wr_tt) <- nameRows(wr_tt)
vars = list( RdWorks = c("Wy", "Wn") , Rain = c("Ry", "Rn"))
print(zmToa <- matrixToMarray(tt = wr_tt, valuenames = vars ) )
```
mFromMarginal *Construct m vector of a bca from marginal probabilities*

Description

Construct m vector of a bca from marginal probabilities

Usage

```
mFromMarginal(
  marg_probs,
  a = 1e-05,
  simple = FALSE,
  min\_prob = 0,
  max\_prob = 2\lambda
```
Arguments

Value

vector of probability masses obtained from uniformly sampling the cut

Author(s)

Peiyuan Zhu

```
x \leftarrow c(2, 2, 1.5, 1.2, 1, 0, 0)mFromMarginal(x, simple=FALSE)
```


Construct a mass vector from qq function.

Usage

mFromQQ(qq, tt)

Arguments

Value

m A corresponding mass vector

Author(s)

Peiyuan Zhu

Examples

tt <- t (matrix(c(1,0,1,1), ncol = 2)) $m < -c(.9,.1)$ cnames <- c("yes","no") x<- bca(tt, m, cnames=cnames) mFromQQ(x\$qq, x\$tt)

Description

Construct a mass vector from qq function and ttmatrix of focal elements recursively.

Usage

mFromQQRecursive(qq, n)

Arguments

Value

m A corresponding mass vector

Author(s)

Peiyuan Zhu

Examples

```
tt<- t(matrix(c(1,0,1,1),ncol = 2))
m < -c(.9,.1)cnames <- c("yes","no")
x<- bca(tt, m, cnames=cnames)
mFromQQ(x$qq, x$tt)
```
mobiusInvHQQ *Mobius inversion of commonality function*

Description

Mobius inversion of commonality function

Usage

mobiusInvHQQ(qq, h)

Arguments

Value

m Mass of the hypothesis

Author(s)

Peiyuan Zhu

```
x \le -\text{bca}(\text{tt} = \text{matrix}(c(0,1,1,1,1,0,1,1,1),\text{nrow} = 3, \text{ byrow} = \text{TRUE}),m = c(0.2, 0.5, 0.3), cnames = c("a", "b", "c"), varnames = "x", idvar = 1)
qq <- commonality(x$tt,x$spec[,2])
mobiusInvHQQ(qq, matrix(c(0,1,0,1,1,0)), nrow = 2, byrow = TRUE))
```


mrf *The Captain's Problem.* mrf*: Relation between variables No Maintenance (M = false) and Repairs at sea (R)*

Description

This dataset is the tt matrix establishing a set of two relations between the two variables. First, Repairs = true if Maintenance = false in $(M \times R)$. We are 20% sure that there will be Repairs if no maintenance. Second, Repairs = false if Maintenance = false in $(M \times R)$. We are 20% sure that there will be no repairs if no maintenance.

Usage

mrf

Format

 $A(0,1)$ matrix with 4 rows and 6 columns.

 $[1,\mathbf{c}(1,2)]$ value = 0, not used

[1,3:6] Identification numbers of the two variables. Column 3,4: variable 6; columns 5,6: variable 8

nospec identification number of the specification

m the value of the specification, a number between 0 and 1

true 1 if true is part of the specification, 0 otherwise

false 1 if false is part of the specification, 0 otherwise

Details

These two relations are implication rules. The remaining mass of 0.6 is allotted to the frame. To construct the tt matrix, we put the variables M and R side by side, as in a truth table representation. Each pair of the subset is described by a row of the matrix as a vector of zeros and ones.

Author(s)

Claude Boivin, Stat.ASSQ

Source

Almond, R.G. [1988] Fusion and Propagation in Graphical Belief Models. Computing Science and Statistics: Proceedings of the 20th Symposium on the Interface. Wegman, Edward J., Gantz, Donald T. and Miller, John J. (ed.). American Statistical Association, Alexandria, Virginia. pp 365–370.

mrt *The Captain's Problem.* mrt*: Relation between variables Maintenance done (M = true) and Repairs at sea (R)*

Description

This dataset is the tt matrix establishing a set of two relations between the two variables. First, Repairs = true if Maintenance = true in $(M \times R)$. We are 10% sure that there will be Repairs if maintenance is done. Second, Repairs = false if Maintenance = true in $(M \times R)$. We are 70% sure that there will be no repairs if maintenance is done.

Usage

mrt

Format

 $A(0,1)$ matrix with 4 rows and 6 columns.

 $[1,\mathbf{c}(1,2)]$ value = 0, not used

[1,3:6] Identification numbers of the two variables. Column 3,4: variable 6; columns 5,6: variable 8

nospec identification number of the specification

m the value of the specification, a number between 0 and 1

true 1 if true is part of the specification, 0 otherwise

false 1 if false is part of the specification, 0 otherwise

Details

These two relations are implication rules. The remaining mass of 0.2 is allotted to the frame. To construct the tt matrix, we put the variables M and R side by side, as in a truth table representation. Each pair of the subset is described by a row of the matrix as a vector of zeros and ones.

Author(s)

Claude Boivin, Stat.ASSQ

Source

Almond, R.G. [1988] Fusion and Propagation in Graphical Belief Models. Computing Science and Statistics: Proceedings of the 20th Symposium on the Interface. Wegman, Edward J., Gantz, Donald T. and Miller, John J. (ed.). American Statistical Association, Alexandria, Virginia. pp 365–370.

This utility function makes use of the valuenames and size parameters of a set of variables to assign values names to the columns of a tt matrix.

Usage

```
nameCols(valuenames, size)
```
Arguments

Value

A character vector of length equal to the sum of the sizes of the variables.

Author(s)

Claude Boivin

Examples

```
infoval <- list(A = c("a1", 'a2"), B = c("b1", 'b2", 'b3"))
sizes \leftarrow c(2,3)
print(nameCols(valuenames = infoval, size = sizes) )
```
nameCols_prod *Naming the columns of the* tt *matrix of a product space*

Description

This utility function makes use of the valuenames and size parameters of a set of variables to assign values names to the columns of the tt matrix of their product space.

Usage

nameCols_prod(valuenames, size)

46 nameRows and the contract of the contract o

Arguments

Value

A character vector of length equal to the product of the sizes of the variables.

Author(s)

Claude Boivin

Examples

```
infoval \leq list(A = c("a1", "a2"), B = c("b1", "b2", "b3"))sizes \leq c(2,3)print(nameCols_prod(valuenames = infoval, size = sizes) )
```


Description

This function determines the name of a row from all the columns of the tt that show 1 for that row.

Usage

nameRows(tt, f)

Arguments

Details

The row containing only 1s is called "frame", to avoid too long a label. The empty set is identified by its code "u00f8". The "+" sign is used to represent the logical "or" operator. The space " " is used to represent the logical "and" operator. Note that in the case of a product space defined on many variables, row labels can become very long.

Value

A character vector of labels obtained for the rows of the tt matrix. The length of the result is nrow(tt).

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Author(s)

Claude Boivin

Examples

```
tt < - matrix(c(0,0,0,1,0,0,0,0,1,1,0,1,1,1,1),ncol = 3, byrow = TRUE)
colnames(tt) <- c("A","B","C")
rownames(tt) <-nameRows(tt)
tt
```
nzdsr *Normalization of a basic chance assignment*

Description

It may occur that the result of the combination of two basic chance assignments with Dempster's Rule of combination contains a non-zero mass allocated to the empty set. The function nzdsr normalizes the result of function dsrwon by dividing the mass value of the non-empty subsets by 1 minus the mass of the empty set.

Usage

 $nzdsr(x, sparse = "no", comm = "no")$

Arguments

Value

z The normalized basic chance assignment.

Author(s)

Claude Boivin, Peiyuan Zhu

References

Shafer, G., (1976). A Mathematical Theory of Evidence. Princeton University Press, Princeton, New Jersey, pp. 57-61: Dempster's rule of combination.

Examples

```
x1 \le -\text{bca}(\text{tt= matrix}(c(1, 0, 1, 1), nrow = 2, \text{ byrow = TRUE}),m = c(0.9, 0.1), cnames = c("yes", "no"),
varnames = "x", idvar = 1)
x2 \le -\text{bca}(\text{tt} = \text{matrix}(c(0,1,1,1),\text{nrow} = 2, \text{byrow} = \text{TRUE}),m = c(0.5, 0.5), cnames = c("yes", "no"),
varnames = "x", idvar = 1)
print("combination of x1 and x2")
x1x2 \le - dsrwon(x1, x2, varname = "x")
nzdsr(x1x2)
# Test with sparse matrices
x1s=x1x2s=x2
x1s$tt <- methods::as(x1$tt, "RsparseMatrix")
x2s$tt <- methods::as(x2$tt, "RsparseMatrix")
x1x2s <- dsrwon(x1s, x2s, use_ssnames = TRUE)
nzdsr(x1x2s)
print("normalization of a bca definition.")
y2 \le -\text{bca}(\text{tt} = \text{matrix}(c(0, 0, 0, 1, 0, 0, 1, 1, 1), \text{nrow} = 3,byrow = TRUE), m = c(0.2, 0.5, 0.3),
cnames = c("a", "b", "c"), idvar = 1)
cat("y2")
cat("\\ '")y2
nzdsr(y2)
```
nzdsrLogsumexp *Normalization of a basic chance assignment with logsumexp*

Description

It may occur that the result of the combination of two basic chance assignments with Dempster's Rule of combination contains a non-zero mass allocated to the empty set. The function nzdsr normalizes the result of function dsrwon by dividing the mass value of the non-empty subsets by 1 minus the mass of the empty set.

Usage

```
nzdsrLogsumexp(x, sparse = "no", comm = "no")
```
Arguments

peeling the second second

Value

z The normalized basic chance assignment.

Author(s)

Claude Boivin, Peiyuan Zhu

References

Shafer, G., (1976). A Mathematical Theory of Evidence. Princeton University Press, Princeton, New Jersey, pp. 57-61: Dempster's rule of combination.

Examples

```
x1 \le -\text{bca}(\text{tt= matrix}(c(1, 0, 1, 1), nrow = 2, \text{ byrow = TRUE}),m = c(0.9, 0.1), cnames = c("yes", "no"),
varnames = "x", idvar = 1)
x2 \le -\text{bca}(\text{tt} = \text{matrix}(c(0,1,1,1),\text{nrow} = 2, \text{byrow} = \text{TRUE}),m = c(0.5, 0.5), cnames = c("yes", "no"),
varnames = "x", idvar = 1)
print("combination of x1 and x2")
x1x2 \le - dsrwon(x1, x2, varname = "x")
nzdsr(x1x2)
# Test with sparse matrices
x1s=x1
x2s=x2
x1s$tt <- methods::as(x1$tt, "RsparseMatrix")
x2s$tt <- methods::as(x2$tt, "RsparseMatrix")
x1x2s <- dsrwon(x1s, x2s, use_ssnames = TRUE)
nzdsr(x1x2s)
print("normalization of a bca definition.")
y2 \le -\text{bca}(\text{tt} = \text{matrix}(c(0, 0, 0, 1, 0, 0, 1, 1, 1), \text{nrow} = 3,byrow = TRUE), m = c(0.2, 0.5, 0.3),
cnames = c("a", "b", "c"), idvar = 1)
cat("y2")
cat("\\ '")y2
nzdsr(y2)
```
peeling *The peeling algorithm*

Description

An implementation of the peeling algorithm based on its description in terms of hypergraphs by R. Almond [1989].

Usage

peeling(vars_def, hgm, hg_rel_names, elim_order, verbose = FALSE)

Arguments

Details

The peeling algorithm works on an undirected graph. Nodes of the graph (variables) are removed one by one until only the variable of interest remains. An order of elimination (peeling) of the variables must be chosen by the user. No algorithm is provided for that matter. At each step, a procedure of extension is applied to the bca's to merge, and marginalization is applied to eliminate a variable. The marginalization has the effect to integrate in the remaining nodes the information of the eliminated variable.

Value

A bca class object.

Author(s)

Claude Boivin

References

• Almond, R. G. (1989) Fusion and Propagation of Graphical Belief Models: An Implementation and an Example. Ph. D. Thesis, the Department of Statistics, Harvard University. 288 pages (for the description of the peeling algorithm, see pages 52-53).

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Examples

```
# Zadeh's Example
# 1. Defining variables and relations
# (for details, see vignette: Zadeh_Example)
e1 <- bca(tt = matrix(c(1, 0, 0, 1, 1, 1), ncol = 2, byrow = TRUE),
m = c(0.99, 0.01, 0), cnames = c("M", "T"),
varnames = "D1", idvar = 1)
e2 \le -\text{bca}(\text{tt} = \text{matrix}(c(1, 0, 0, 1, 1, 1), \text{ncol} = 2, \text{byrow} = \text{TRUE}),m = c(0.99, 0.01, 0), cnames = c("C", "T"),
varnames = "D2", idvar = 2)
p\_diag \leftarrow bca(t t = matrix(c(1,1,1), ncol = 3, byrow = TRUE),m = 1, cnames = c("M", "T", "C"),
varnames = "D", idvar = 3)
# Defining the relation between the variables
# tt matrix
tt_r1 <- matrix(c(1,0,1,0,1,0,0,1,0,1,0,0,0,1,
1,0,0,1,1,0,0,1,0,0,1,0,1,0,0,1,1,0,0,1,0,
0,1,1,0,0,0,1,0,1,0,1,0,1,0,1,1,1,1,1,1,1),
ncol = 7, byrow = TRUEcolnames(tt_r1) = c("M", "T", "C", "T", "M", "T", "C")
# The mass function
spec_r1 \leq -matrix(c(rep(1,7), 2, rep(1,7), 0), ncol = 2, dimnames = list(NULL, c("specnb", "mass")))# Variables numbers and dimension of their frame
info_r1 \leq matrix(c(1:3, 2,2,3)), ncol = 2, dimnames = list(NULL, c("varnb", "size")) )
# The relation between e1, e2 and a patient p
r1 \leq - bcaRel(tt = tt_r1, spec = spec_r1, infovar = info_r1,
varnames = c("D1", "D2", "D"), relnb = 1)
# 2. Setting the incidence matrix of the grapph
rel1 <- 1*1:3 %in% r1$infovar[,1]
ev1 <- 1*1:3 %in% e1$infovar[,1]
ev2 <- 1*1:3 %in% e2$infovar[,1]
meddiag_hgm <- matrix(c(ev1,ev2, rel1), ncol = 3,
dimnames = list(c("D1", "D2", "D"), c("e1","e2", "r1")))
# 3. Setting the names of the variables and their frame of discernment
meddiag_vars1 <- c(e1$valuenames, e2$valuenames, p_diag$valuenames)
# 4. Names of bca specifications (evidence and relations)
meddiag_rel_names <- c("e1", "e2", "r1")
# 5. Order of elimination of variables
elim_order \leq c(1,2,3)tabresul(peeling(vars_def = meddiag_vars1, hgm = meddiag_hgm,
hg_rel_names = meddiag_rel_names, elim_order = c(1, 2, 3)) )
```
plautrans *Plausibility transformation of the singletons of a frame*

Given a mass function defined on some subsets of a frame Θ, the application of the plausibility transformation to the singletons of Θ yields the probability distribution associated with this mass function.

Usage

plautrans(x)

Arguments

x A bca mass function.

Details

We compute the plausibility measure of all the singletons of the frame of discernment. The probability distribution of the singletons is obtained from their plausibility measures.

Value

The matrix of singletons with their plausibility transformation added in the last column.

Author(s)

Claude Boivin

References

Cobb, B. R. and Shenoy, P.P. (2006). On the plausibility transformation method for translating belief function models to probability models. Journal of Approximate Reasoning, 41(3), April 2006, 314–330.

```
x \le -\text{bca}(\text{tt} = \text{matrix}(c(0,1,1,1,1,0,1,1,1),\text{nrow} = 3,byrow = TRUE), m = c(0.2, 0.5, 0.3),
cnames = c("a", "b", "c"),varnames = "x", idvar = 1)
plautrans(x)
```
This utility function takes the input matrix of a relation between two or more variables and yields its product space representation.

Usage

productSpace(tt, specnb, infovar)

Arguments

Value

The matrix of the product space representation of the relation.

Author(s)

Claude Boivin

```
ttfw <- matrix(c(1,0,1,0,0,1,0,1,1,1,1,1), nrow = 3,
 byrow = TRUE,
 dimnames = list(NULL, c("foul", "fair", "foul", "fair")) )
specfw <-c(1,1,2)infovarfw \leq matrix(c(5,7,2,2), ncol = 2,
dimnames = list(NULL, c("varnb", "size")) )
rownames(ttfw) <- nameRows(ttfw)
ttfw
productSpace(t t = t t f w, speech = spec f w, infovar = infovar f w)
```


This utility function is used to obtain a summary of a vector of data for many operators. The function is taken from the project APL in R (<https://rpubs.com/deleeuw/158476>).

Usage

reduction(x, f)

Arguments

Value

The result of applying the chosen operator to all the elements of the vector is an object of length 1.

Author(s)

Claude Boivin

References

- Jan de Leeuw and Masanao Yajima (March 07, 2016) *APL in R (Version 009)*, Source code. <https://rpubs.com/deleeuw/158476>
- G. Helzer. (1989): *An Encyclopedia of APL*, second edition, I-APL LTD, St. Albans, G.B.
- L. Gilman and A. J. Rose.(1974): *APL an Interactive Approach*, Second Edition, John Wiley, New-York.

```
reduction(c(1, 2, 3, 4), f = " -")reduction(c(1,0,1,1,0), f = " |")reduction(c("a", "b", "c"), f = "paste")
```
shape 55

Description

shape returns the dimension of given array or returns the length of a given vector. The function is taken from the project APL in R (<https://rpubs.com/deleeuw/158476>).

Usage

shape(a)

Arguments

a An array or a vector.

Value

The dimension of the array a or the length of the vector a.

Author(s)

Claude Boivin

References

- Jan de Leeuw and Masanao Yajima (March 07, 2016) *APL in R (Version 009)*, Source code. <https://rpubs.com/deleeuw/158476>
- G. Helzer. (1989): *An Encyclopedia of APL*, second edition, I-APL LTD, St. Albans, G.B.
- L. Gilman and A. J. Rose.(1974): *APL an Interactive Approach*, Second Edition, John Wiley, New-York.

```
shape(array(c(1:6), c(2,3)))
shape(c("a", "b"))
```
swr *The Captain's Problem.* swr*: Relation between variables Sailing delay (S), Weather at sea (W), and Repairs at sea (R)*

Description

This dataset is the tt matrix establishing a relation between S, W and R, where $S = 0:3$, $W = (f \text{oul})$, fair) and $R =$ (true, false). The goal of this relation is to account for other causes of sailing delay. All the elements (s, w, r) of $(S \times W \times R)$ where W or R is true add one day of sailing delay. We put a mass value of 0.9 to this subset. The remaining mass of 0.1 is allotted to the frame.

Usage

swr

Format

An integer matrix with 6 rows and 10 columns.

 $[1,\mathbf{c}(1,2)]$ value = 0, not used

[1,3:10] Identification numbers of the three variables. Column 3 to 6: variable 3; columns 7,8: variable 7, columns 9,10: variable 8

nospec identification number of the specification

m the value of the specification, a number between 0 and 1

3 1 if 3 is part of the specification, 0 otherwise

2 1 if 2 is part of the specification, 0 otherwise

1 1 if 1 is part of the specification, 0 otherwise

0 1 if 0 is part of the specification, 0 otherwise

foul 1 if foul is part of the specification, 0 otherwise

fair 1 if fair is part of the specification, 0 otherwise

true 1 if true is part of the specification, 0 otherwise

false 1 if false is part of the specification, 0 otherwise

Details

To construct the tt matrix, we put the variables S, W, R side by side, as in a truth table representation. Each triplet of the subset is described by a row of the matrix as a vector of zeros and ones.

Author(s)

Claude Boivin, Stat.ASSQ

tabresul 57

Source

Almond, R.G. [1988] Fusion and Propagation in Graphical Belief Models. Computing Science and Statistics: Proceedings of the 20th Symposium on the Interface. Wegman, Edward J., Gantz, Donald T. and Miller, John J. (ed.). American Statistical Association, Alexandria, Virginia. pp 365–370.

tabresul *Prepare a table of results*

Description

This utility function is a more detailed version of the belplau function. Different tables of measures of belief, plausibility and of the plausibility ratio can be obtained, namely by removing subsets with zero mass if present, or by asking for singletons only. Unlike function belplau, function tabresul does not reconstruct the row names from the column names. You can assign short rownames of your choice to the tt matrix of your resulting bca before calling function tabresul.

Usage

```
tabresul(x, singletonsOnly = FALSE, removeZeroes = FALSE)
```
Arguments

Value

A list of two elements:

- mbp The table of focal elements with the addition of their associated mass, degree of belief, plausibility and the plausibility ratio.
- con The measure of conflict between subsets.

Author(s)

Claude Boivin

```
x \le -\text{bca}(\text{tt} = \text{matrix}(c(0,1,1,1,1,0,1,1,1),\text{nrow} = 3,byrow = TRUE), m = c(0.2, 0.5, 0.3),
{\text{cnames}} = c("a", "b", "c"),varnames = "x", idvar = 1)
y \le - bca(tt = matrix(c(1,0,0,1,1,1),nrow = 2,
byrow = TRUE), m = c(0.6, 0.4),
cnames = c("a", "b", "c"), varnames = "y", idvar = 1)
```
58 ttmatrix and the control of the control

```
xy \leftarrow dsrwon(x, y)xyNorm <- nzdsr(xy)
tabresul(xyNorm)
## print("Show all elementary events")
xy1 <- addTobca(nzdsr(dsrwon(x,y)),
matrix(c(0,1,0,0,0,1),
nrow = 2, byrow = TRUE))
tabresul(xy1)
## print("Remove focal elements with 0 mass")
tabresul(xy1, removeZeroes = TRUE)
print("Retain singletons only")
tabresul(xy1, singletonsOnly = TRUE)
```
ttmatrix *Construct a description matrix from a list of subsets names.*

Description

Construct a description matrix from a list of subsets names.

Usage

 $ttmatrix(x, sparse = "no")$

Arguments

Value

ttmat A corresponding logical description matrix

Author(s)

Claude Boivin

```
subsets_names <- list(c("b", "c"), "b", c("a", "b", "c"))
ttmatrix(subsets_names)
znames <- list("empty", "a", c("b", "c"), c("a", "b"), c("a", "b", "c") )
print(ttmatrix(znames) )
print(ttmatrix(znames, sparse = "yes") )
```
ttmatrixFromMarginal *Construct tt matrix of a bca from marginal probabilities*

Description

Construct tt matrix of a bca from marginal probabilities

Usage

```
ttmatrixFromMarginal(
 marg_probs,
 from_above = FALSE,
  simple = FALSE,
 min\_prob = 0,
 max\_prob = 2)
```
Arguments

Value

matrix of possible subsets obtained from the cuts

Author(s)

Peiyuan Zhu

```
x \leftarrow c(2, 2, 1.5, 1.2, 1, 0, 0)ttmatrixFromMarginal(x, FALSE)
```


Construct a description matrix from qq function.

Usage

```
ttmatrixFromQQ(qq, n, valuenames)
```
Arguments

Value

ttmat A corresponding logical description matrix

Author(s)

Peiyuan Zhu

Examples

```
tt<- t(matrix(c(1,0,1,1),ncol = 2))
m < -c(.9,.1)cnames <- c("yes","no")
x<- bca(tt, m, cnames=cnames)
ttmatrixFromQQ(x$qq,as.integer(x$infovar[1,2]), unlist(x$valuenames))
```
ttmatrixPartition *Create partition matrix*

Description

Create partition matrix

Usage

ttmatrixPartition(n, m)

ttmatrixPartition 61

Arguments

Value

h binary partition matrix of size n by m

Author(s)

Peiyuan Zhu

```
# test singleton hypotheses
x \le -\text{bca}(\text{tt = matrix}(c(1,1,0,1,1,1), \text{ nrow = 2, byrow = TRUE}), m = c(0.8, 0.2), \text{ cnames = } c(1,2,3))pa <- ttmatrixPartition(x$infovar[2], x$infovar[2])
belplau(x, h=pa)
```
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