$$
\begin{aligned}
A = \lim_{n\to\infty} U &= \lim_{n\to\infty} \sum_{i=0}^{n-1} (\Delta x \cdot f(a + i \cdot \Delta x) \\
&= \lim_{n\to\infty} (\Delta x \cdot f(a) + \Delta x \cdot f(a + \Delta x) + \Delta x \cdot f(a + 2 \cdot \Delta x) + \Delta x \cdot f(a + 3 \cdot \Delta x) + \ldots + \Delta x \cdot f(a + (n-1) \cdot \Delta x)) \\
&= \lim_{n\to\infty} \Delta x \cdot (f(a) + f(a + \Delta x) + f(a + 2 \cdot \Delta x) + f(a + 3 \cdot \Delta x) + \ldots + f(a + (n-1) \cdot \Delta x)) \\
&= \lim_{n\to\infty} \Delta x \left(a^2 + (a + \Delta x)^2 + (a + 2 \cdot \Delta x)^2 + (a + 3 \cdot \Delta x)^2 + \ldots + (a + (n-1) \cdot \Delta x)^2\right) \\
&= \lim_{n\to\infty} \Delta x \left(a^2 + \left(a^2 + 2a\Delta x + (\Delta x)^2\right) + \left(a^2 + 2 \cdot 2a\Delta x + 2^2 (\Delta x)^2\right) + \left(a^2 + 2 \cdot 3a\Delta x + 3^2 (\Delta x)^2\right) + \ldots\right. \\
&\qquad \left. + \left(a^2 + 2 \cdot (n-1)a\Delta x + (n-1)^2 (\Delta x)^2\right)\right) \\
&= \lim_{n\to\infty} \Delta x \left(na^2 + 2a\Delta x (1 + 2 + 3 + \ldots + (n-1)) + (\Delta x)^2 \left(1^2 + 2^2 + 3^2 + \ldots + (n-1)^2\right)\right) \\
&= \lim_{n\to\infty} \Delta x \left(na^2 + 2a\Delta x \frac{n(n-1)}{2} + (\Delta x)^2 \frac{n(2n-1)(n-1)}{6}\right) \\
&= \lim_{n\to\infty} \frac{b-a}{n} \left(na^2 + 2a\frac{b-a}{n}\frac{n(n-1)}{2} + \left(\frac{b-a}{n}\right)^2 \frac{n(2n-1)(n-1)}{6}\right) \\
&= \lim_{n\to\infty} \frac{b-a}{n} \left(na^2 + a(b-a)(n-1) + \frac{(b-a)^2}{n}\frac{(2n-1)(n-1)}{6}\right) \\
&= \lim_{n\to\infty} (b-a) \left(a^2 + a(b-a)\frac{n-1}{n} + \frac{(b-a)^2}{n^2}\frac{(2n-1)(n-1)}{6}\right) \\
&= \lim_{n\to\infty} (b-a) \left(a^2 + a(b-a)\left(1 - \underbrace{\frac{1}{n}}\right) + (b-a)^2 \frac{1}{6}\left(2 - \underbrace{\frac{3}{n}} + \underbrace{\frac{1}{n^2}}\right)\right)
\end{aligned}
$$

jeweils Nullfolgen für $n \to \infty$